Measurement and the mathematics underachiever

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Difficulties learning measurement concepts and skills include comprehending;

- the attribute (for example, area, perimeter, volume); use physical actions or drawings.
- relational terms applied to particular attributes,
- measurement as matching or comparing with a standard; use physical matching, stepping off
- meaning of the lines on a scale; space between two lines represents a tangible physical quantity
- a quantity can be described in different ways by converting between scales of measurement
- fractional quantities in measurement.

A task analysis of length skills

- use the attribute of length in free play activities with terms such as long, short, little
- use longer, shorter, etc., to relate two lengths,
- sort more than two lengths into those that are longer / shorter than a particular length,
- match or cover a particular length by a multiple
- Answer "How many (of these blocks) exactly match this (longer length) ?
- Produce a length that is a certain number of blocks long.
- Decide whether two lengths are equal when direct length matching is not possible.
- Construct and use a length scale of centimetres or blocks, up to 20 cm long initially.
- Extend this scale to the metre and use the longer scale.
- Describe the need for the metre, describe a length in both centimetres and metres.
- Convert between metres and centimetres.
- Describe the need for the millimetre, modify the scale to include fractions of a cm. The skill to use this scale and to describe a measurement in terms of mm and cm.

Area concepts and skills

- Distinguish between area and perimeter by using actions
- Introduce area as How many are needed to cover this space ?
- (3) Different shapes have the same area. Games such as "Area-21", Tesselations.

Measurement formulae

- Derive each formula from several specific instances, using actions
- use drawings, actions etc. to highlight the property to be calculated.
- when showing how the formula is derived, encourage actions suing cardboard cut-outs etc.

Mathematics underachievers learning spatial knowledge

What do we mean by spatial knowledge ? Consisting of two aspects

(1) shapes, for example, the number of sides they have, the number and size of angles, and
(2) position or location in space, for example, above, horizontal, etc.

The types of spatial difficulties displayed by underachievers

- difficulties dealing with directional concepts such as up and down, right and left.
- difficulties with the elementary spatial concepts such as 'inside', 'above' and 'under'.
- a spatial concept is associated with inappropriate criteria.
- the child cannot 'act mentally' on a shape or visualize it being changed or transformed
- the concept of angle and the extent of rotation cause difficulty.
- the child uses inappropriate perceptual features to categorize shapes
- the child has difficulty representing 3-dimensional objects in 2-dimensions.

Why do children form misconceptions about spatial concepts ? Misconceptions about spatial concepts can be attributed to a number of sources, for example

- perceptual difficulties integrating the parts of a spatial stimulus to form the whole or discriminating between the main visual information and irrelevant background information,
- lack of earlier sensory-motor experiences, such as building, matching, shape manipulation.
- difficulties learning visually or tactually

- inadequate teaching.

How is spatial knowledge acquired ? : Van Hiele's theory --> five levels of development:

(1) Level 1 - shapes are distinguished in their overall or global appearance and not on the basis of relationships between the number or length of sides or angles.
(2) Level 2 - an awareness of parts of shapes begins to develop, as children play with shapes in various ways they note individual properties of particular shapes.
(3) Level 3 - children begin to organize Level 2 findings into relationships between shapes. They generalize - as the number of sides a shape has gets bigger, so does the number of angles, that every square is also a rectangle and that every rectangle is also a parallelogram, that four-sided shapes can be made from two triangles and that five sides can be made from three triangles.
(4) Level 4 - children develop 'child-propositions' to deduce one spatial property from another, for example, if the sum of the angles in a triangle is 180 degrees then it follows that if the total number of degrees in a shape is 180 degrees, the shape must be a triangle.
(5) Level 5 - children learn the more abstract aspects of deductive reasoning to prove geometric relationships. This is the theoretical level of understanding spatial concepts.

Language plays a role in learning. Each level uses a vocabulary to represent the concepts and relationships. The following sequence moves pupils from the direct instruction to the student's understanding independent of the teacher;

(1) inquiry; the teacher engages pupils in two-way discussions about the spatial ideas, guides them to construct an understanding of the topic being studied.
(2) directed orientation; the teacher sequences activities for guided pupil exploration, leading them to become familiar with the characteristic structures.
(3) explication; the students build on their foregoing experiences to refine their comprehension of the topic being examined and express their ideas and understandings.
(4) free orientation; students develop their own procedures for solving longer, more complex spatial problems, identify many of the relations between the spatial ideas being learnt.
(5) integration; the students review their findings and form an overview. The relationships are unified into a new domain of thought.

A sequence for teaching spatial knowledge.

Students who have difficulty learning spatial knowledge can be assisted by the following sequence:

(1) students manipulate shapes in free play situations, such as building, solving spatial problems, shape post-boxes, drawing 2- and 3-D objects explore shapes through physical actions, etc.
(2) students recognize and name individual shapes.
(3) students analyse the characteristic properties of individual simple regular shapes on the basis on the number of sides and angles.
(4) students manipulate groups of shapes, describe shapes from different perspectives, fit shapes together,
(5) students generate more general spatial concepts by physical actions, such as
    (a) all squares are rectangles
    (b) the set of polygons.

(6) students recognize spatial concepts in different perceptual context, for example, they act on one shape to produce the other and discuss the effect of particular transformations. Gradually they are encouraged to visualize these types of actions.