**Mathematical giftedness and talent**

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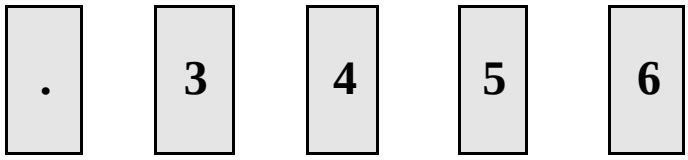
## Thinking creatively in mathematics

Work through each of the following maths tasks. Keep track of the knowledge you use and the ways of thinking you use to solve each. As you go through these tasks, keep a record of your self talk, that is, what you say to yourself as you work through it.

***Place value*** Five cards.

You are given these cards. Your task is to make each of the numbers below. You can use some or all of these cards




What is the smallest number you can make using the cards ?

What is the largest number you can make ?

What numbers between 4.3 and 6 can you make ? Say as many as possible.

Which of the following number is nearest to 4.5 ? 4.4 4.56 4.6

You need to explain how to decide which number is closest to a particular number ? How would you explain it.

| What knowledge did you use ? | What thinking did you use ? |
|---|---|
| | |

This task is intended to help students to think creatively about place value.

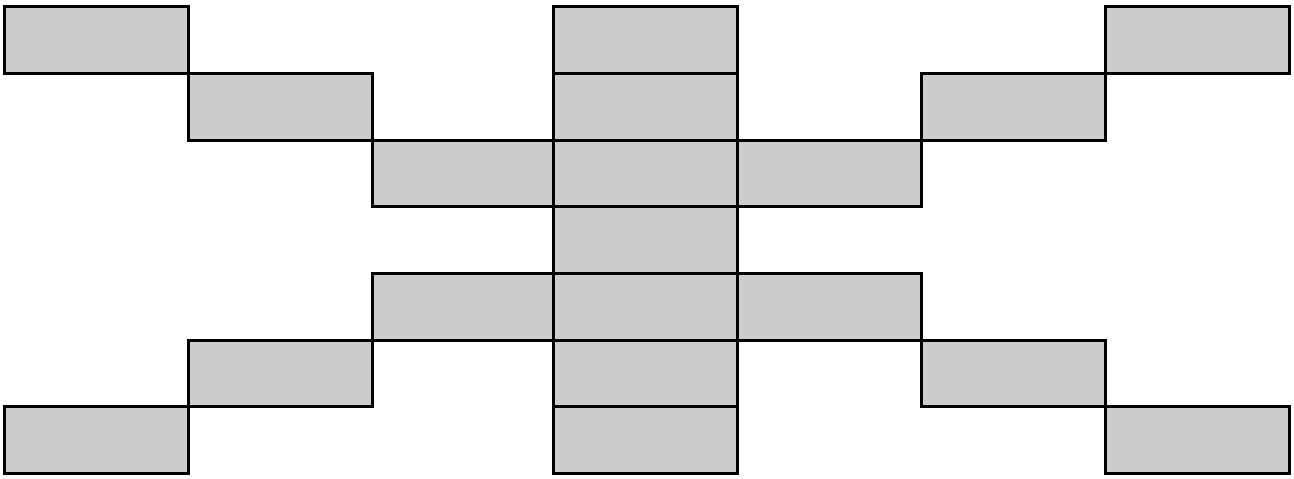
The aim of the tasks here are to give students the opportunity to think creatively about topics that are aspects of regular maths curricula. It would be easy to give students gimmicky tasks that are novel and unusual. Often, however, these tasks have nothing to do with regular maths concepts.

Our aim is to give the students tasks that will enhance both their knowledge of maths concepts and their awareness of ways of thinking creatively in mathematical ways. Our focus is on how gifted students learn maths and how we can design teaching to match this. Whether we help gifted students learn maths or not doesn't depend on what tasks we use but

- how we use them
- what we have the students do
- how they change their thinking and learn new ways of doing this.

We need to evaluate the tasks we use both for teaching and for assessing learning. A task may be creative and interesting but it may have no relevance to the particular topic we are teaching. The world is full of gimmicks. We need to be selective in the tasks we use.

***Counting tasks.*** Suppose you have 19 tiles Write a number from 1 to 19 on each tile.
How can you arrange them in the following way so that the 3 arms add up to 76 ?



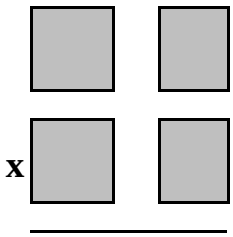
Metroplana is the city of the future. Town planners have decided to build houses so that they have maximum access to parkland. Houses are built on one side of each street only. Each street has more than one house. If there are 4 houses in a street, the numbers 1 to 4 are used but not in that order. If there are 5 houses in a street, the numbers won't be in order, that is, they won't go in the order 1, 2, 3, 4, 5.

In Adam St no house numbers are in order. What is the smallest amount of houses that could be in the street ?

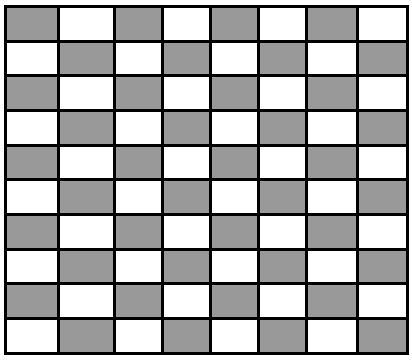
In Beulah Street, no house numbers are in order. As well, no house numbers are different by 2, that is, you would not find house with number 5 beside the house with number 3 or the house with number 7. Again, what is the smallest amount of houses in Beulah St ?

***Computation tasks*** Suppose you have these number cards. How could you arrange these cards in the following multiplication task to get an answer than 6000 ?

| **1** | **4** | **7** | **9** |
|---|---|---|---|




***Spatial tasks.*** Someone said there are 204 squares on a chess board. Was this person right ?



Robert's birthday cake is in the shape of a perfect cube. It is totally covered with chocolate icing. When some of his friends saw it they said "Chocolate brings us out in a rash". Robert asked how many friends wanted a piece that

- had no icing
- had icing on one side only
- had icing on two sides
- had icing on three sides

Then he did a quick calculation. "Could you cut the cake into cube sized bits so that there are eight times as many pieces with no icing as the number of pieces that have icing on three sides ? " he asked his mother. That will give everyone the piece they would like. How many friends were there at his party ?

***Measurement tasks*.** Tom needs to fill 6 10-litre buckets as quickly as he can. He can use 4 taps that are next to each other. Each tap delivers 1 litre or water each minute. What is the shortest time he can do this in ?

***Algebra tasks*** Find all integers m and n that satisfy the equation $4m^2 + 4mn - 3n^2 = 29$. Suggest three other equations that can be solved in this way.

**Defining and identifying students gifted and talented in the domain of Mathematics**

**What is mathematical talent ?** Mathematical talent refers to an unusually high ability to understand mathematical ideas and to reason mathematically, rather than just a high ability to do arithmetic computations or get top grades in mathematics.

**Case study : Tom** 5 years old

- "You know Dad, you will only be 54 when your age is twice mine!"
- intrigued by numbers and numerical relationships since he was very small; he liked to count things and organize groups of objects, fascinated by calendars, telephone numbers, dates, ages, measurements, and almost anything else dealing with numbers
- played with and manipulated the information he was learning; examine each idea and eagerly search to discover new, interesting, and unusual relationships and patterns
- little formal instruction in mathematics, at the age of 5 he has an incredible amount of mathematical knowledge and is amazingly sophisticated in using this knowledge to discover new ideas and solve problems.

**Mathematically gifted vs good at school maths** Not all students who achieve the highest test scores in mathematics class are necessarily highly talented in mathematics.

- focus on developing computational skills, conformity to taught procedures little opportunity to demonstrate the complex types of reasoning skills characteristic of truly talented students.
- test scores and grades of less able students who are good in computation, attentive in class, willing to help, and conscientious about completing assignments carefully in the prescribed manner will often be as high as the test scores and grades of students who are genuinely talented in mathematics.

Need more than high achievement in school to indicate high ability in mathematics.

**Identifying mathematical giftedness:** common features include

- insight into arithmetical problems that require careful reasoning, ask questions that target the conceptual basis of ideas
- more efficient reasoning, see short cuts, dead ends, makes 'big leaps' in thinking, may not write down every step, learn rapidly
- superior reasoning powers,
- generalise and transfer knowledge well beyond other students to new, untaught math situations.
- outstanding problem solving
- flexible reasoning, see options, prepared to change mind, unusual ability to think and work with mathematical problems in flexible, creative ways rather than in a stereotypic fashion
- sustain concentration, maintains interest and grips a challenge, persistent intellectual curiosity
- excellent communication

- see implications
- remembers what is learnt, may remember minor ideas learnt earlier
- creative thinking ability; see problems from different perspectives, suggest original, not taught ways of solving problems, combine taught knowledge in creative ways
- initiative; doesn't wait to be guided or led through task but wants to manage and direct own progress, use own knowledge or set own agenda for gathering information and guidance
- gets excitement and pleasure from intellectual challenges, genuinely intrinsically motivated to learn maths, intense curiosity about numeric information, do not value extrinsic motivation
- keen observational skills
- sets high standards
- unusually quick in learning, understanding, and applying mathematical ideas.
- high ability to think and work abstractly and the ability to see mathematical patterns and relationships.
- extract formal structure from context of a mathematics problem and operates within that structure.
- switch flexibly from one approach to another and identifies and avoids non productive approaches.
- operate with symbols and spatial concepts.
- quickly recognize similarities, differences, and patterns.
- visualize and interprets facts and relationships.
- achieve clarity, simplicity, economy, and rationality in arguments.

Three main aspects
- "willingness to work hard," encompasses a variety of characteristics, including being focused, committed, energetic, persistent, confident, and able to withstand stress and distraction
- "natural mathematical ability," and
- "highly creative; capable of divergent thinking and of combining their experience and skills from seemingly disparate domains to synthesize new products or ideas.

Those
- labelled as mathematical geniuses may be thought of as those who innately possess most of these characteristics, but are not necessarily willing to work hard or are not very creative.
- with a high degree of mathematical ability, creativity, and willingness to work hard may be labelled "truly gifted" and need minimal outside influence to develop their abilities.
- in the double and triple intersections of these three characteristics are mathematically gifted.

**Systematic process to identify /select mathematically talented students**

***Intelligence Tests*** IQ test results, by themselves, not useful indicating mathematical talent. This is a specific aptitude, while an IQ score is a summary of many different aptitudes and abilities.

***Creativity Tests*** : mathematical talent is not always apparent in creativity test results.

***Mathematics achievement tests*** can assist to identify high ability in mathematics, but their results have to be interpreted carefully: they

- are often computation-oriented and don't show how a student reasons mathematically

- seldom have enough difficult problems to appropriately assess the upper limits of a talented student's ability or show that this ability is qualitatively different from that of other very good, but not mathematically talented, students.

Given these limitations, use the results of nationally normed achievement test such as the Iowa Tests of Basic Skills. Students scoring at or above the 95th percentile on the Composite or Math Total may have high ability in mathematics, but more information is needed to separate the high achievers from the truly gifted. Mathematically talented students scoring below the 95th percentile will have to be recognized from other clues.

***Mathematics Aptitude Tests***; used in basically the same way that the results of achievement tests are used. Select students scoring above the 95th percentile on a mathematics aptitude test. They have some of the same limitations as achievement tests except that, because they place less emphasis on computation and more on reasoning skills, the results from these tests are often more useful in identifying mathematically talented students.

***Gifted Characteristics Scale*** s the Gifted Evaluation Scale (GES) (McCarney, 1987) to identify gifted students. Subscale scores in the areas of: Intellectual, Creativity, Specific Academic Aptitude, Leadership Ability and Performing and Visual Arts.

Dimensions of Self-Concept Scale (DOSC) (Michael, Smith, & Michael, 1989) a self-report inventory of five school-related factors of self concept in the areas of: Aspiration, Anxiety, Academic Interest and Satisfaction, Leadership and Initiative, and Identification vs. Alienation.

***Out-of-Grade-Level Mathematics Aptitude Tests***

- a test designed for and normed on students two to five grade levels above the grade placement of the student (Assouline & Lupkowski-Shoplik, 1997) with more difficult items to allow students to demonstrate their mastery of more advanced concepts.
- gives a much better assessment of mathematical reasoning skills because the student must find ways to solve problems, many of which he or she has not been taught to do.
- spread out the scores of those students who had been clustered at the top of the grade-level test
- have many difficult problems that will challenge even the most capable students, thus making it possible to discriminate the truly talented from others who are just very good in mathematics.
- indicate the degree of acceleration and/or enrichment needed by the talented students.
- use only with students who already have demonstrated strong mathematics abilities on regular-grade-level instruments or those who show definite signs of high mathematics ability.

The out-of-grade-level testing procedure has been used successfully in several mathematics talent searches and school mathematics programs with junior and senior high school students over the past 15 years. More recently, there have been programs that have successfully used the procedure in the elementary grades.

Figure 1. Identification Checklist

| Name | Ability test | Ach test | Gifted | Creative | Teacher-Parent Nominations | Out-of-Grade Level Test |
|---|---|---|---|---|---|---|
| John | 97 | | yes | yes | | |
| Sally | | 95 | yes | | | |
| | | | | | | |
| | | | | | | |

Common features that seem to be important ingredients in the mathematics programs of mathematically talented students.

- Allow mathematically talented students to work together to learn mathematics. They learn from each other, reinforce each other, and help each other over difficulties.
- The program stresses mathematical reasoning and develops independent exploratory behavior. Discovery learning, looking for underlying principles, engaging in special projects in

mathematics, problem solving, discovering formulas, looking for patterns, and organizing data to find relationships.

- The program de-emphasize repetitious computational drill work and cyclical review.. As ability in mathematics increases, the benefits to be gained from this type of activity decrease.

- The scope of the curriculum is extensive, to provide an adequate foundation for students who may be mathematicians in the future.

- The program should be flexibly paced. Students are placed at an appropriate instructional level on the basis of an assessment of their knowledge and skill and are allowed to progress at a pace limited only by his or her ability and motivation. Flexible pacing can be achieved in the following ways:
    - Continuous progress. Students receive appropriate instruction daily and move ahead as they master content and skill.
    - Compacted course. Students complete two or more courses in an abbreviated time.
    - Advanced-level course. Students are presented with course content normally taught at a higher grade.
    - Grade skipping. Students move ahead 1 or more years beyond the next level of promotion.
    - Early entrance. Students enter elementary school, middle school, high school, or college earlier than the usual age.
    - Concurrent or dual enrolment. Students at one school level take classes at another school level.

Framework for evaluating maths tasks in terms of their relevance for gifted and talented students.

| | Do you search for any particular pattern(s) car numbers or post-codes? | Do you explain physical phenomena with an original math model? |
|---|---|---|
| involve convergent vs divergent thinking | convergent | mainly divergent thinking |
| that is popular / unusual | popular | unusual |
| of high / low quality | high quality | high quality |
| leads to a popular / unusual product | popular | unusual and |
| leads to a high / low quality product | high quality | high quality product |
| activity is intrinsically / extrinsically motivated | intrinsically motivated | intrinsically motivated |
| reflects high / low task commitment | mild task commitment | high task commitment |
| high / low initiative, | mild initiative | high initiative |
| high / low degree of intensity of doing it | mild degree of intensity | very high intensity |
| activity is done with low/high frequency, | frequently | infrequently |
| for brief / high time duration, | longer time duration | long time duration |
| is/is not focused on one specific field in maths | one specific field | one specific field |
| theoretical level of creative talent of mathematics | mild | profound level |
| % of students who do it | | 1% - 2% of adolescents |
| level of creative talent in mathematics | mild | profound level |

**Modifying school mathematics to cater for gifted and talented learners**

Learning characteristics of gifted and talented learners learning

| Maths teaching frequently | Gifted students often | Teaching implications |
|---|---|---|

| | | |
|---|---|---|
| • requires students to learn abstract procedures, sometimes without access to meaning initially.<br><br>• requires students to learn to write ideas in a particular symbolic way, setting out ideas in an organised way.<br><br>• teaches ideas with a focus on detail, small increments at a time.<br><br>• involves working through pre-determined sets of exercises with little opportunity to vary this. | • learn quickly and readily, recall ideas easily but may have difficulty with rote recall of math knowledge<br><br>• show carelessness in routine tasks.<br><br>• learn in holistic way initially.<br><br>• require fewer repetitions of and less exposure to an idea to learn it. | • teach maths knowledge eg tables first with a meaningful base.<br><br>• teach students how to read and write maths symbolic statements.<br>• allow students to show what they know in preferred ways first and translate these into symbolism.<br>• introduce general aspects of new ideas and help them identify and 'slot in' the detail. Draw network diagrams of the developing ideas.<br>• vary how these students work through repetitive tasks |
| • assumes external motivation, emphasis is not on developing intrinsic interest | • learn by being curious and motivated to 'want to know'. They are 'self-driven'. | allow students to take some aspects of the learning in directions that interest them |
| • is externally directed and teacher directed and managed; the focus is on internalising external knowledge.<br>Ideas developed at a rate determined by the teacher. There is often little encouragement to question or analyse the ideas. Rather, they are encouraged to accept the ideas. | • tend to learn spontaneously without direct teaching, are independent learners and prefer to manage and direct their own learning and ask a range of questions that extend their knowledge. They become bored and frustrated if the learning pace is too slow and may find learning difficult in situations in which their learning is directed, authoritarian, and those in which their curiosity is not challenged.<br><br>• may become behaviour problems in more directed, closed learning contexts or in repetitive tasks. | • allow students to direct and monitor their won learning, to plan their way through tasks, monitor how well they are progressing.<br><br>• allow students to indicate when they think they have mastered an idea<br><br>• provide open-ended tasks, allow students to make up tasks for each other, to debate ideas, discuss when they can use them, etc.<br><br>• encourage them to have a 'map of their journey through Grade 5 maths". They can manage their journey, see themselves making progress. |
| • is presented in an teacher-class context, instructor directs learning. | • often prefer to work by themselves and may be irritated by class peers who do not understand the ideas at the same depth.<br><br>• may lack confidence in interaction with peers, have difficulty understanding and valuing the learning of others. | • allow students to learn maths though co-operative learning activities, eg jigsaw.<br><br>• develop an awareness of group learning. |
| • develops a topic at the same level of complexity for all students (eg, factorise, how to add fractions) with little opportunity for individual students to develop a deeper understanding of the ideas | • have a wide general knowledge that may be commensurate with that expected of older pupils and an extreme knowledge in areas of interest. They can transfer and generalize their knowledge well. | • activate student knowledge of a topic at the outset, for example, *"What do you know about percentages ? What questions can you ask about them ?"*<br>• ask them to suggest areas in which they can use the knowledge and to invent unusual uses for the ideas, unusual tasks based on the ideas. |

| | | |
|---|---|---|
| • teaches pre-determined 'ways of thinking mathematically, teaches 'bits' of thinking at a time and builds on earlier ways of thinking; 'you need to learn the set of steps for completing the task'<br><br>• is convergent; the thinking is towards convergent outcomes. There is often little encouragement of open-ended outcomes and interpretations.<br><br>• focuses on thinking analytically in specific ways about small parts of ideas at a time.<br><br>• focuses on learning established concepts and procedures in an internalising way | • think and make decisions quickly in personal creative rather than taught ways and often think so quickly that they have difficulty saying how they went about solving a problem.<br><br>• can cope with several ideas at once and enjoy inventing or designing ideas, put ideas together in unexpected ways and may give unexpected responses to questions. They may skip steps in expected thinking sequences.<br><br>• often reason synthetically and analytically in complex ways about maths ideas and see issues that are not developed by the teaching.<br><br>• can recognize and use patterns and relationships at an advanced level. | • encourage them to verbalise their personal solution strategies and then ask *"What is the most efficient way of doing this ?*"<br>• teach them self-instruction strategies for working their way through tasks systematically.<br>• encourage and value open-ended comments and questions about ideas being learnt.<br>• allow students to share personal impressions of ideas with others; *How are they like/ different from other people's ideas ?*<br><br>• introduce new ideas in global, open-ended ways rather than "*This is what you do*"<br><br>• provide students with 'raw data' problems and encourage them to abstract procedures. |
| • develop small tasks at a time | • can concentrate on tasks of interest for prolonged periods and show a high level of perseverance. | • provide more detailed, research-oriented tasks on occasions. |
| • teaches through 'successive approximations', learners need to be prepared to 'change their minds', take risks, and not be concerned by making errors as they align their ideas with the maths ideas. | • set high standards and goals for themselves, judge themselves harshly and may worry about expectations that they should be 'perfect' and yet know that they aren't. They may have difficulty 'risk-taking' in learning, be more anxious, put stress on themselves and feel stress from others due to unrealistic expectations | • model how to use errors to sharpen an idea; making an error means 'having another go'.<br>• ensure that errors are corrected; *You are on the way / almost right*.<br>• errors help you see the bits you need to work on more.<br>• negative consequences do not necessarily follow from errors and don't mean you will do badly on future test.<br>• help students monitor their work to detect errors, that it is fine to change their minds, to recognise possible 'danger areas', and how they can deal with these. |
| • provides positive feedback and valuing for convergent outcomes and ways of thinking | • may feel different and alienated from peers, sometimes because they don't get positive affirmation from peers and teachers, feel they have less in common with peers, (their peers may not comprehend the ideas they are communicating and they feel that there is something wrong with them) may have difficulty communicating with their peers, and seem 'the odd one out'. | • have a 'positive feedback' policy and schedule, help these students get positive feedback for what they know. |

**Additional teaching procedures that relate to *how we teach maths*, rather than what we teach :**

- model how to ask questions, how to be curious about school maths, validate students asking questions, give students words for curiosity and inquisitiveness, build it into regular dialogue in teaching
- begin each lesson with students presenting their own problems
- teach higher level meaning ways of thinking about maths; teach the meaning aspects of maths ideas, for example

| ways of thinking in maths | Examples |
|---|---|
| describe quantities in different ways | We use different ways of describing a quantity to achieve different purposes such as<br>• factorising • expanding<br>• simplifying • doing indices. |
| 'get a situation ready' to do a maths operation. | • finding the lowest common denominator<br>• factorising a quadratic to solve it. |
| using symbolism | the values of this, how this allows us to describe, simulate, infer. The assumptions made by the symbolism |
| using graphical representation | The types of questions you can ask about graphical representations, the assumptions they make. How you infer mathematically from a graph, the questions you can ask, for example, the trend in the relationship, how the relationship changes, how far you can apply it. |

- teach algorithms in a meaningful way, for example, to teach students how to factorise 2x + 6, for gifted students, have them
    - decide what the solution / end product would be like- intuit the solution ; 12 -----> 2 x 6; we need to write 2x + 6 as two terms multiplying. Guess at what it might be.
    - construct the factorising procedure as self instruction
    - have students examine how far they can transfer it.

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