TEACHING SENIOR MATHS WITH CAS: MAJOR ISSUES FOR CURRICULUM, ASSESSMENT AND TEACHING

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The possibility of a CAS-active curriculum provides an exciting opportunity to rethink approaches for teaching and learning in the senior mathematics curriculum. During 2001 students and teachers in three schools have been involved in the trial of Mathematical Methods (CAS) Units 1&2 and their experience provides a starting point for discussions in schools that may be able to offer Mathematical Methods (CAS) in the future.

Implementing a new CAS subject

For the pilot study, Mathematical Methods (CAS), students are able to use calculators equipped with a computer algebra system (HP 40G, Casio FX 2.0 and TI 89) for all aspects of the course, including examinations. This paper will outline some issues and challenges facing the schools that are currently implementing this new subject and provide guidelines for schools invited to participate in the expanded pilot program. Following a successful conclusion to the pilot program, it is envisaged that all schools will be able to choose to implement either MM or MM(CAS) or both of these. The pilot study design for MM(CAS) is available on the VCAA website (HREF1). This paper highlights some of the challenges and rethinking that is required for implementation of a CAS-active curriculum. It is intended to promote professional dialogue about approaches for improving student mathematical understanding when CAS is available.

Implementation of MM(CAS) will require careful planning in the year prior to offering the subject to students. In this year there will be a reasonable amount of administrative work that arises normally in the formal process of introducing a new VCE subject. Figure 1 outlines the administrative tasks that schools may need to undertake. These include tasks such as deciding on the number of classes, gathering resources and producing a course outline. It is crucial that this administration does not become the main focus of the year and that the process of professional development of staff and discussion of approaches for teaching with CAS commence as soon as possible and become the focus of the implementation.

This "implementation" phase, shown in Figure 1, is beneficial in helping staff develop their basic calculator skills and also in raising some broader issues associated with CAS calculator use. CAS calculators are highly sophisticated instruments and learning to use them well requires time. The introduction of CAS into the senior curriculum will impact on the work of all teachers of mathematics, not just those teachers of senior mathematics. Teachers in earlier years should be encouraged to trial related activities, where appropriate, with their classes (see, for example the Mathematics CSF II, pp 154 and 212). Encourage the involvement of all teachers in discussions about CAS use to enable all staff to reflect on how their use may improve student mathematical understanding. In the introductory stage of CAS use, it is likely that teachers will focus on trialing "one-off" activities with their classes and consideration of the impact of CAS could be the focus of discussion at mathematics faculty meetings. As teachers become more adept at CAS use, and trial more

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classroom activities, they will hopefully move to a situation where they consider the broader curriculum and pedagogical changes possible with CAS. Note that while CAS calculators incorporate the features of graphics calculators, they also provide many more features such as symbolic manipulation and algebraic solution of equations.

Rethinking teaching

The introduction of a CAS-active curriculum poses challenges in a study where a significant focus has been the attainment and practice of "by-hand" skills. Figure 2 suggests some issues that could be discussed when a school offers MM (CAS). The introduction of the new course may require a "rethinking" of approaches to teaching and learning and we believe that it is essential to carefully consider these prior to offering the course, as well as reflecting upon changes in practice as the course is being taught.

There are differences in the curriculum underpinning MM(CAS) and MM and it is important that teachers are aware of these. For example, in schools that offer both MM and MM(CAS), the "by-hand" expectations for these courses and associated assessments will be different. The MM(CAS) course outline states explicitly the reduced 'by- hand' skills required of students. This needs to be communicated effectively to students, particularly when their peers may be studying a non-CAS course. However fluent CAS use also requires broad algebraic insight (see Pierce & Stacey, 2001), beyond a narrow interpretation of by-hand skills. "By-hand" facility is required to develop understanding , rather than calculation in its own right. Students need to be clear about teachers' expectations and the requirements of the study design. Be explicit about expectations and ensure that they reflect the study design and advice from the VCAA. Use the sample examination questions available from the VCAA website (HREF3) as a guide to what is expected of students. Consider carefully the wording of questions and the solutions that are given as suitable models for written solutions for MM(CAS) questions and tasks as students will need guidance about how they should record their working. Ball & Stacey (submitted) give guidelines about how students written records may change – see the CAS-CAT website for some ideas.

In comparison to MM, the availability of CAS for MM(CAS) enables a more general treatment of content. For example, the new study design includes consideration of polynomial functions of degree *n*, rather than only to degree 4. It also enables inclusion of more real life applications of mathematics. This is reflected in the areas of study containing a wider coverage of material and a greater emphasis on applications of mathematics. This shift in emphasis requires pedagogical changes and can provide new opportunities for approaching questions and topics, but teachers will need to think carefully about how they can incorporate CAS into their teaching to achieve these aims. To achieve this requires teachers to be relatively familiar with some aspects of the technology initially (for example, the algebra and graphing menus) and a willingness to explore new features of the CAS as they learn about the technology. For teachers, familiarity seems necessary in order to feel confident in answering student questions and holding class discussions about various approaches for solving problems. These discussions should also include consideration of the limitations of the technology and strategies to help students become discerning users of CAS. Helping teachers become familiar with the technology will most likely be the first area where mathematics faculties wanting to implement a CAS subject will start.

The teachers we are working with have found that access to CAS has enabled them to explore more ideas in their classrooms than was previously the case when only graphics calculators were available. They see this as an extremely positive aspect of the new course and have discussed changes in their approach to topics, such as the increased use of functional notation through the defining function of a CAS calculator (see for example, Stacey, Ball, Asp, McCrae & Leigh-Lancaster, 2000). One approach is to define a function as $f(x)$ at the start of a problem and then use function notation throughout. For example, if a student had to substitute the value $x = 3$ into a cubic function, they could define the cubic as $f(x)$ and then determine $f(3)$ or $f(x + h)$. Access to CAS can enable a more algebraic approach to topics and faculties should consider how this could impact on approaches to particular topics. There could, for example, be an increased emphasis on the use of parameters (see Stacey et al, 2000). Discussions should focus on teaching and learning opportunities available in a CAS curriculum, rather than only the situation where classes use CAS for a short task.

Teachers in the CAS-CAT research project did not feel that teaching students how to use particular calculator functions impinged greatly on their curriculum time. Building on their knowledge of graphics calculators, students developed different approaches for remembering how to use particular functions of the CAS calculators. Some students maintained their own summary notes, some wrote calculator steps beside questions, while others preferred summaries of steps provided by teachers. We believe that whatever method is preferred, CAS use is necessarily much more than a sequence of button pushes and there must be evident links to the mathematics used to solve a problem.

ADMINISTRATIVE (FOR CURRENT PILOT SCHOOLS ONLY)

Submit proposal to appropriate school body to offer Mathematical Methods (CAS)

- For the pilot MM(CAS) study the VCAA must have invited your school to be involved

Obtain a copy of the pilot MM(CAS) study design ,sample exam questions and supplementary questions from VCAA website

- Ensure that all mathematics staff have a copy of the pilot study design
- Discuss the difference between MM(CAS) and Mathematical Methods, including sample examination questions, as a faculty

Select a CAS calculator

- If you are happy with it, a good starting point is the same brand as the graphics calculator currently used.

Purchase CAS calculators as a faculty resource

- You may decide to distribute some of these to your maths staff so that they can become familiar with them
- Use the others to trial some activities and develop classroom experience

Teacher resources

- Every MM(CAS) class needs an overhead projector and projection unit

Select a suitable textbook

- There may be limited reference, or no reference, to CAS so additional notes will likely be needed in the initial stages of using CAS

Notify students about offering MM(CAS)

- Prepare entry for VCE handbook and possibly a presentation at VCE information evening

Determine the number of classes that your school will offer

- Offering more than one class enables teachers to work together and discuss approaches for teaching
- Consider staffing for the new subject. Initial teachers will need commitment and your support.

Purchase of CAS calculators

- Decide if students will purchase their own CAS calculators or whether the school will provide them

Course outline

- Produce a course outline being aware that the time allowed for, and sequence of, topics may be different to previous years – be flexible and allow for review

Timetable Issues

- Should MM and MM(CAS) be blocked together?
- What student movement will be allowed between MM and MM (CAS) classes, and when ?

Information for parents

- Decide if there will be a parent information evening to supplement information in the VCE handbook

IMPLEMENTATION

Staff learn basics of use

- Devise a professional development strategy to help familiarize staff with some of the basics and then help them become more advanced users.
- Use both internal and external professional development

Develop and trial lessons across the school

- Provide opportunities for staff to report back about their experiences
- Discuss approaches for managing this technology in the classroom, e.g. Year 9 solving equations, Year 11 Differentiation, Year 8 Developing the notion of "function"
- Discuss the nature of a "good" activity and the new activities that are possible

Collect as many resources as possible for staff

- Provide access to professional reading and classroom based resources. Try our CAS-CAT website(HREF2).

Discuss approaches to topics with CAS

- Have staff trial new approaches to topics and report back to faculty meetings

Incorporate CAS into existing school coursework assessment tasks

- Write a task to be completed by Year 11 Mathematical Methods using CAS. Evaluate approaches.

Figure 1: Guidelines for schools planning for Mathematical Methods (CAS)

Students are expected to use mathematical notation, but there may be some calculator syntax that is acceptable notation in working. For example, when showing that the value $x=2$ has been substituted into the function $f(x)=4x^3$ students may choose to write $f(2)$ or $4x^3|\ x=2$, indeed the latter form is similar to existing conventions such as $dy/dx|_{x2}$ However, students should not record button sequences such as press F2 as this has no relation to the mathematics being used to solve the problem. It could be helpful for mathematics faculties to discuss the mathematical and calculator notation that they are going to use so that there is some consistency across the school. Linked to this is forming an expectation of what students may record for their solutions when the CAS calculators carry out many of the routines that would previously be completed "by-hand". This is an area where we are expecting significant changes to evolve.

The balance between "by-hand" skills and CAS use is an issue that needs to be addressed. In all work, teachers should be providing students with clear guidance about what "by-hand" work needs to be shown and what may now be done routinely using CAS. There will be procedures that have been allocated a considerable amount of practice time in non-CAS courses that are now require little practice when students have access to CAS. Students should be able to make judicious choices as to when it is appropriate to carry out certain routines by hand or by using CAS. As a consequence, the focus can move from routine application of a procedure to interpretation of a mathematical result. The teachers we are working with are beginning to consider these issues and the importance of having discussions among the teachers implementing a CAS course is unquestionable. It is also likely that views will change as teachers work more with CAS and they become more familiar with how the technology may be used to assist student mathematical learning.

RETHINKING TEACHING

How can CAS help students?
What pedagogical changes are required in a CAS classroom?
What strategies will be used to help students become familiar with basic operations of CAS?
- Will students be provided with handouts or be expected to write their own notes?
- When students record button sequences, CAS syntax or mathematical notation?
- Will students complete a unit on calculator use or learn about capabilities as they go along?

What are the elements of the curriculum where "by-hand" skills should be de-emphasised now that CAS is available?
What are the new possibilities for approaches to topics and assessment tasks?
What "by-hand" skills do students need when they have a CAS? What sorts of tasks will demonstrate achievement of these skills? What sorts of tasks demonstrate the value of these skills?
How do you help students and staff to become good CAS users? Are there some students who will need extra help?
How should students approach problems when they have CAS? We think there will be some major differences (for example, see Stacey & Ball, 2001). For example, a powerful strategy will be to begin questions by defining a function.
When should students use CAS and when should they do something "by-hand"?
Will some calculator syntax be acceptable as well as mathematical notation?

Figure 2: Some issues in implementing MM(CAS)

Strategic planning for school based development of a CAS-active senior mathematics curriculum: a broader view.

Thinking about the development of a proposal for a CAS-active senior mathematics curriculum, and the process for facilitating the effective consideration of such a proposal by a school's executive and curriculum committee and other key stakeholders (as appropriate) needs to incorporate the following elements:

- A clear articulation of the context in which the proposal is framed, for example international perspectives on the use of technology, in particular CAS, and related research literature. This may take the form of a collection of discussion and/or briefing papers.
- Government policies, directions and initiatives at state and federal levels, for example, the current state governments '*Reality Bytes*' (State of Victoria, 2001).
- A summary of the systemic policies with respect to the given technology (CAS in this instance) for example, current possible use of the technology in coursework assessment (HREF4) the VCAA MM(CAS) pilot study (HREF1).
- A philosophical framework within which discourse mathematics education takes place for example, the *public educator* perspective, as articulated in Ernest (1991).

- Clear links between school based principles and policies for curriculum development, such as the schools mission statement or charter, in particular aims and, goals or objectives related to technology.
- A rationale for the proposal in terms of the pedagogical and curriculum goals, directions and interests for the school's mathematics department, teachers and students.
- Identification of resource and professional development implications.
- Formulation of a possible implementation strategy.
- Development of a strategy for monitoring and reviewing related processes and outcomes.
- Dissemination to, and communication with, stakeholders and interest groups, for example, the school council and parent groups.

The stimulus leading to the development of such a proposal often arises from the interest and enthusiasm of a teacher or group of teachers, who in conjunction with the head of faculty or mathematics coordinator, initially seek background information and ideas from a variety of sources. These may be articles or papers published in system, teacher association or mathematics education bulletins and journals (for example, the *VCE Bulletin*, *Vinculum* and AAMT's *Australian Senior Mathematics Journal*, MAV and MERGA Conference handbooks) conference sessions, mathematics texts that utilize a given technology such as CAS, or discussions with other teachers, mathematicians, or mathematics educators who have used CAS. Teachers in the mathematics department can then work together to develop a consensus, or common understanding, on how a response to this stimulus might be developed, leading to a formal proposal for consideration by the school executive and curriculum committee, or similar.

This might occur in the context of an opportunity arising for participation in a pilot implementation program. For example, in one school that forwarded an expression of interest for involvement in response to an article on the VCAA expanded MM(CAS) pilot implementation program (*VCE Bulletin*, April 2001), staff had developed a general awareness that calculators with CAS capabilities provide a further stage of technological development from graphics calculators, and were willing to look at new technologies that have the potential to support mathematics learning in new ways. Several staff were also aware of related technology developments in other countries. A consensus view was developed by the mathematics faculty to express interest in the expanded pilot program, and subsequently endorsed by the head of the mathematics faculty. This was then put forward as a formal proposal to the school's curriculum board (which votes on such proposals) and subsequently approved by this board.

Conclusion

In parallel with the various administrative and other tasks associated with preparing for the implementation of a CAS-active senior mathematics course such as the pilot MM(CAS) study, mathematics faculties can consider more generally how CAS could be used to improve student mathematical understanding. Preparation of the new VCE subject has drawn upon expertise and advice from the school and tertiary sectors, so that the study design provides a consensus position on what is important. However, we believe that schools can take advantage of this opportunity to rethink possible approaches to teaching that would be supported by access to CAS and also to promote discussion of and a refreshed commitment to the mathematical skills and knowledge to be achieved by the completion of a senior mathematics course.

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HREF1

http://www.vcaa.vic.edu.au/vce/studies/MATHS/caspilot.htm - CAS pilot, Victorian Curriculum and Assessment Authority, Victoria.

HREF2

http://www.edfac.unimelb.edu.au/DSME/CAS-CAT/ - CAS-CAT project, Department of Science and Mathematics Education, University of Melbourne, Victoria.

HREF3

http://www.vcaa.vic.edu.au - Victorian Curriculum and Assessment Authority, Victoria.

HREF4

http://www.vcaa.vic.edu.au/VCE/STUDIES/ASSESS/PDFS/Mathmeth.pdf - Victorian Curriculum and Assessment Authority