CAS, CALCULUS AND CLASSROOMS

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Abstract

Three teachers helped design and then taught an experimental program of introductory calculus in which students had full access to calculators with a computer algebra system (CAS) in the classroom, at home and during tests. Each class obtained similar mean scores on the test. However they made very different use of the CAS and performed very differently on items. One class frequently used the CAS. The second preferred by-hand algebraic techniques. The third group of students, with weaker algebraic skills, used CAS more selectively and demonstrated good understanding built from illustrating algebraic ideas graphically. The study demonstrates how teachers "privileging" impacted on student learning.

Introduction and Background

Computer algebra systems (CAS), incorporating graphical, numerical and symbolic algebra capabilities, have much to offer in the teaching of calculus. By reducing the obstacle of manipulative algebraic skills, they can release time in calculus courses to spend on concept development, problem solving and investigations (Hillel, 1993). In addition, the capacity to provide multiple representations and the ability to move freely between them makes CAS a teaching tool to enhance conceptual development. Several experimental studies now support these expectations. For example, Heid (1988), O'Callaghan (1998) and Repo (1994) have claimed that students using a CAS developed better conceptual understanding although they differ about whether this is accompanied by a loss of computational skills. There is also some research evidence to support the belief that students become better problem solvers while using a CAS (Heid, 1997). The availability of symbolic algebra on calculators, rather than on standard computers only, and the attendant reduction in unit price has stimulated even more interest in teaching calculus with CAS. Cappuccio (1996), for example, describes specific ways to teach calculus using the TI-92, a Texas Instrument calculator which incorporates a CAS similar to the desktop DERIVE.

The study reported in this paper is part of a larger study conducted with colleagues Barry McCrae and Gary Asp which explored the implications for state-wide assessment of teaching and learning calculus with CAS. In this paper we describe how three different classes of students used CAS in different ways while learning introductory calculus using the same teaching program. As such it is a study of the type recommended by Penglase and Arnold (1996, p.79) “which directly attempt[s] to address the issues of graphics calculator use within particular learning environments.”

The data reported here show how the outcomes for students in the three classes were quite different and we attempt to link these to the personal style and philosophy of each teacher. A wide variety of factors may affect the impact of an innovation such as new technology in the classroom. Amongst these are teacher-related socio-cultural factors such as attitude towards technology, prior experience teaching with technology and personal beliefs about the ways students should be taught (Thomas, Tyrrell, & Bullock, 1996). Of particular interest in this paper is Wertsch’s concept of “privileging” which Berger (1998) describes as “the social setting and values which may elevate one form of mental functioning over another and in this way privilege a particular form of mental operation such as algebraic or graphical reasoning (p.19).”

The teaching trial, research methods and data collection

Early in 1998 a team including McCrae, Asp, the first author and three volunteer teachers designed a twenty-lesson introductory calculus program that aimed to use CAS to enhance conceptual understanding, connections between representations and appropriate use of the technology. Subsequently the teachers each taught the program to one class. In total there were 59 students in three Year 11 classes (students aged approximately 17 years) from two schools. The three classes had similar distributions of mathematical ability, but Class C had the weakest algebraic skills. All the students were familiar with the TI-83 graphical calculator. For this study, they were given a TI-92 graphics calculator for use during lessons, at home and for most testing. The three teachers were experienced in teaching mathematics with the TI-83 graphical calculator but inexperienced with the TI-92.

Students regularly completed questionnaires, challenging questions and log sheets to describe their feelings and progress with the calculus and technology. All students completed a written test (sample items below) and for each item they indicated whether they had used the calculator. Seventeen students were given a task-based interview by the first author, who also observed approximately half of the lessons and maintained a journal. The teachers wrote a brief reflective evaluation after each lesson and completed questionnaires and Likert items before and after the program.

Results

Table 1 shows the mean scores for each class on the written test (maximum possible score was 70). Although the absolute scores seem low, the results were very pleasing because of the difficulty of the items for Year 11 students. One third of the written test items were taken from state examination papers for Year 12 students and the study classes performed well in comparison to Year 12 classes.

Table 1 shows that the three classes performed similarly overall. However, the item-by-item analysis in Tables 2 and 3 shows differences. Table 2 indicates the relative amount of calculator use. For each class, it shows the number of items where that class made the greatest or least percentage use of the calculator. For example, on 25 items, use was made of the calculator by a greater percentage of students in Class A than in Class B or C. Clearly Class A students used the calculator to a much greater extent than Classes B and C. Class B students used it least.

Table 1. Mean scores and standard deviation for total test by class.

	Class A (N=15*)	Class B (N=18*)	Class C (N=13*)
Mean	24.1	26.7	27.9
Standard Deviation	9.4	12.0	10.3

** Not all students participated in all tests, so the numbers in the Tables vary.*

Table 2. Number of items where each class made greatest and least use of the calculator.

	Class A (N=15)	Class B (N=19)	Class C (N=16)
Greatest % calc use	25	3	5
Least % calc use	5	17	11

Table 3. Number of attempted items correct and incorrect by calculator use and by class.

	Class A (N=15)		Class B (N=19)		Class C (N=16)	
	calc	no calc	calc	no calc	calc	no calc
Correct	152	61	156	124	147	101
Incorrect	111	86	78	129	47	92
Total	263	147	234	253	194	193

Table 3 shows the total number of items attempted by students in each class broken down by calculator use or non-use and by success. This item-by-item analysis confirms the results in Table 2 that Class A students chose to use the CAS most frequently (64% of items attempted) and Class B least (48%), slightly less than Class C (50%). The Table 3 analysis shows more difference in overall success than does Table 1, with Class A correct on 52% of items attempted, Class B 57% correct and Class C 64% correct. This shows that there is no simplistic conclusion that greater use of CAS leads to better results. Class C achieved a higher success rate on items attempted both with and without the calculator. The percentages of items correct where students had used the calculator were 58% (Class A), 67% (Class B) and 75% (Class C) and without the calculator 41% (Class A), 49% (Class B) and 52% (Class C). This indicates that Class C made best strategic use of the CAS calculator, as will be explored in the next sections.

Conceptual and procedural errors

Despite the program's emphasis on conceptual development using technology, many students had difficulty with fundamental concepts, particularly rates of change and distinguishing between the gradients of secants and tangents. Test responses were analysed and errors were classified as conceptual and procedural. We classified as conceptual errors those which occurred when understanding was not demonstrated or the process to use was not correctly formulated. This is similar to Orton's (1983) definition of structural errors which "arose from some failure to appreciate the relationships involved in the problem or to grasp some principle essential to solution" from Donaldson in 1963. Procedural errors were algebraic, graphical, and numerical in origin or related to incorrect use of the calculator. Conceptual errors cannot be eliminated with CAS use; procedural errors may be avoided with CAS use. On a group of items attempted by all but a few students, the conceptual error rate per student was 7.3 for Class A, 5.7 for Class B and 4.9 for Class C. However, the procedural error rate was similar for all classes despite varying CAS use: 2.3 per student for Class A, 2.4 for Class B and 2.8 for Class C. In the CAS environment, procedural errors were less frequent than conceptual errors.

Success on specific item types

Many of the test items were grouped for analysis into "*core*", "*symbolic*" or "*options*" groups. There were approximately 5 items of each type. *Core* items were characterised by high conceptual and low procedural demands. In these items, using CAS is no advantage. One example is an item that gave the graph of a function and asked the student to select the graph of the derivative from five possible graphs. CAS would not be useful as no symbolic representation of the function was given.

Symbolic items have high demands on algebraic procedures and low conceptual demands. In these items, the symbolic manipulation capability of the calculators could be useful. One example on the written test was "Find dy/dx for $y = x^3(2x+1)^2$ giving your answer in factorised form". With CAS, an item like this is nearly trivial, but without it, remembering the differentiation rules and the algebraic manipulation can be difficult.

The *options* items are those that present the student with a choice between graphical and algebraic approaches. Using CAS may be advantageous. The following example item was accompanied by a diagram which gave a graph of the equation over a relevant domain:

> Given that a rider on the track of a super roller coaster follows the curve with equation $y = 1/720(x^3 + 20x^2 - 1200x)$, find the maximum height above the ground reached by the roller coaster.

This problem is an *options* item because it can be solved algebraically or graphically. The graphical route is conceptually easier than the algebraic pathway, which requires several conceptual formulation steps. With CAS the algebraic and graphing procedures are easy but not without it.

Taking the results for *core* items as indicating conceptual understanding shows that Class C students demonstrated the highest level of understanding (53.9% of approximately 80 attempts were successful) and Class A the least (40.0% of attempts were successful). We believe that the high score on these *core* items in Class C was due to the way the teacher demonstrated concepts in both graphical and algebraic terms. In total there were only 13 attempts to use the calculator (inappropriately) on these items, mostly from Classes A and C and they were mainly unsuccessful, as would be expected.

In contrast, on the *symbolic* items many students sensibly chose to use CAS. Class A had the highest CAS use and was most successful. Classes B and C both under-utilised CAS, making unnecessary mistakes in algebraic manipulation. In the interviews, students from Class A and B showed greater proficiency than students from Class C in using the algebraic facility on the calculator. For example, to find a gradient at a point, over half of the students interviewed from Classes A and B used a one-line instruction to differentiate the function and substitute the value. In contrast, nearly half of students from Class C attempted a graphical approach.

The *options* items could be solved algebraically or graphically. More Class B and C students used the calculators here than for the *symbolic* items. However, Class C students more often used the graphical facility in contrast to the algebraic route favoured by Class B students. This observation is supported by the student interviews. For example, on an *options* item used in the interviews, a graphical approach was selected by one out of five Class A students interviewed and by one out four Class B students but by 3 out of 5 Class C students. Confirming the previous result about choice of CAS, all 5 Class A students used the calculator, whereas half of the 4 Class B students worked by hand.

The differences between the behaviours of the classes that have been derived from the analysis above are summarised in Table 4. The terms used in the table express relative achievement only. For example, "higher" is relative to the other classes and is not an absolute judgement.

Table 4. Summary of the behaviours of the three classes in the study.

Behaviour	Class A	Class B	Class C
Use of calculator	most frequent	least frequent	frequent
Decision to use calculator	too frequent	discriminating	discriminating
Preferred approach	algebra by calculator	algebra by hand	graphical
Algebraic proficiency	moderate by hand	higher by hand	lower by hand
Graphical skills	lower	moderate	higher
Procedural competence	good	good	good
Conceptual understanding	lower	moderate	higher

Classroom influences

Teacher predictions

Prior to the teaching trial the teachers used a Likert scale to describe their own students' competence and probable reaction to using the new calculators. Table 5 shows their ratings. Again, the table entries express relative positions rather than absolute judgements. The table is highly consistent with the profile of each class that emerged by analysing the test results above. In particular, Class C has an orientation to graphical approaches and Class B to algebraic approaches.

Table 5. Teachers' predictions about class abilities and reactions.

Class characteristic	Class A	Class B	Class C
Algebraic competence	moderate	higher	moderate
Graphical competence	moderate	moderate	higher
Reaction to new technology	likely to succeed	will probably succeed	very likely to succeed

Teaching styles

During the teaching of the CAS program, about half of the lessons were observed by the first author and these observations lead to the descriptions summarised in the first half of Table 6. Table 6 also reports responses to three key indicators from a set of Likert-type items completed by the teachers (with additional written comments) at the end of the program. We summarise these observations as follows. Teacher A had a very positive attitude to technology, encouraged his students to use it as often as possible and gave priority to algebraic strategies. Students were taught efficient calculator procedures for standard tasks. Teacher B preferred the traditional algebraic approach using graphs when essential. He emphasised by-hand algebra, being wary that students might not otherwise develop adequate skills. Teacher C encouraged his students to use both algebraic and graphical methods and to explore connections between them. His explanations used the links between representations.

Discussion

The mean scores for each class on the written test were very similar and a casual observer would assume that learning outcomes for all students were similar. In fact this is not the case. During the teaching trial, the students in each class had very different cognitive experiences evidenced by the different ways the calculator was used. Each class acquired different conceptual understandings, a different set of competencies and different abilities to discern whether or not it would advantageous to use the various features of the calculator. These differences can be linked to the different teaching styles, personal philosophies and cognitive preferences of the teachers, even though they all helped plan the program and were thoroughly aware of

Table 6. Comparison of teacher characteristics

Characteristics	Teacher A	Teacher B	Teacher C
Classroom observations			
Teaching style	Direct instruction	Guided discovery	Guided discovery
Direction of lesson	Followed lesson plan	Controlled exploration	Open exploration
Attitude to using CAS	Enthusiastic	Reserved	Enthusiastic
Structured lesson around use of calculator	Mostly	Sometimes	Mostly
Used algebraic explanations	Very often	Very often	Often
Used graphical explanations	Sometimes	Sometimes	Often
Used both algebraic and graphical	Rarely	Rarely	Often
Teachers' own perceptions			
My usual teaching style suited the CAS Calculus project	Agree Θ	Disagree #	Agree *
There was little formal emphasis on by-hand skills and using pen and paper techniques	Agree	Disagree	Less emphasis, not little
Students enjoyed learning calculus while using CAS	Most yes, some enthusiastic	Some yes, some no	Most yes, a few no

Θ"I did not find it difficult to incorporate the changes" #"I found it difficult to monitor student work"
*"I have already used graphic calculator technology (especially with the view screen) for demonstration in classes."

the aims of the project and taught with the same guidelines. Teacher A privileged technological and algebraic approaches. Teacher B privileged conceptual understanding and by-hand algebraic approaches and Teacher C privileged graphical approaches and conceptual understanding built from illustrating algebraic ideas graphically. As a consequence, Class C's conceptual error rate was lowest. These students understood what to do in algebraic contexts so they could compensate for poor algebraic skills by appropriate use of the calculator and by substituting algebraic with graphical procedures whenever possible.

As we noted in the introduction, many authors have drawn attention to the great potential of CAS in providing multiple representations of mathematical concepts and objects. Multiple representations provide opportunities for students to employ different methods of problem solving and support students finding individual ways of understanding. However, the capacity to provide multiple representations will also lead to greater diversity of teaching methods. The three teachers in our study intended to teach the same material in the same way, according to the lesson guidelines that the whole team prepared. Yet the implementations of the lesson guidelines varied significantly and the differences translated into substantial differences in how their students solved problems and what they understood.

This research raises two questions for our future research. Firstly, the teachers judged their students' abilities and attitudes to technology quite accurately. Did they adapt their teaching practices to take into account the abilities of their students or did they project their own mathematical preferences onto the students and teach in accordance with these? Secondly, the mean scores on the test were very similar, so is there a privileging or teaching style that should be recommended to teachers? Perhaps each particular privileging enhances development in certain directions and constrains it in others. What privileging (if any) constitutes the most advantageous learning environment for students?

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